Anisotropic four-wave mixing in planar LiNbO$_3$ optical waveguides

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We report on the generation of an efficient phase-conjugate wave by anisotropic four-wave mixing in planar iron-doped y-cut LiNbO$_3$ waveguides. The pump waves are orthogonally polarized with respect to the signal and phase-conjugate waves. Reflectivities of the phase-conjugate signal of as much as 65% are reached in the waveguide. Experimental results are compared with numerical solutions of the corresponding system of coupled-wave equations.

Photorefractive LiNbO$_3$ crystals are suitable media for frequency-degenerate two- and four-wave mixing, which permits amplification and phase conjugation of coherent light beams.$^{1,2}$ Besides the widely investigated nonlocal nonlinearity that is due to diffusion of photoexcited carriers,$^2$ photovoltaic currents excited by circularly polarized light$^4$ also permit efficient steady-state energy transfer and phase conjugation.$^5$

Four-wave mixing in waveguides has been studied in doped glasses$^6$ and polymers.$^7$ In this Letter we report what is to our knowledge the first anisotropic generation of an efficient phase-conjugate wave in a planar waveguide. The pump waves are orthogonally polarized with respect to the signal and phase-conjugate waves, and thus the interaction is called frequency-degenerate anisotropic four-wave mixing. Each pair of orthogonally polarized beams writes a holographic grating driven by photovoltaic currents, and the beams can interact through this grating and in addition parametrically through the grating written by the other pair of waves. The coupled-wave equations for this process are given, and experimental data are compared with theoretical results.

In LiNbO$_3$ crystals, redistribution of photoexcited charge carriers is mainly caused by the photovoltaic effect. Following from a phenomenological theory,$^4$ the polarization-dependent photovoltaic current density is given by

$$j = \sum_{i,m} (\beta_{0lm}^\epsilon + i \beta_{0lm}^\kappa) E_i^* E_m.$$  

Here $\beta_{\epsilon,\kappa}$ are the real linear (symmetric) and circular (antisymmetric) photovoltaic tensor components and $E_{i,m}$ are the interacting light fields.

The photovoltaic current causes the buildup of a periodic space-charge field $E_s$, which leads to a perturbation of the dielectric tensor through the electro-optic effect, $\Delta \epsilon_{ij} = -\epsilon_{ii} r_{ijk} E_s E_{jk}$, where $r_{ijk}$ is the electro-optic tensor component. The perturbation $\Delta \epsilon$ has unshifted components according to the tensor elements $\beta^\epsilon$ and components shifted by $\pi/2$ according to $\beta^\kappa$, relative to the isophase surfaces with a phase difference of the interacting light fields $E_{i,m}$ of $2p\pi$ with $p = 0, 1, 2, \ldots$. As is well known,$^5$ the shifted grating leads to an energy exchange between the two interacting beams (two-beam coupling).

Let us now consider the interaction of an extraordinarily (TE-) polarized signal wave 1 propagating along the +x axis and two ordinarily (TM-) polarized counterpropagating (+x and -x) pump waves 2 and 4 in a planar y-cut LiNbO$_3$ waveguide (Fig. 1). As has been shown$^8$ in detail, in this configuration only orthogonally polarized beams traveling in the same direction can write a grating. For counterpropagating beams there is practically no direct interaction, thus reflection gratings (e.g., $k_3 - k_1$) can be neglected here. The waves 1 and 2 write a phase grating with wave vector $K = k_3 - k_1$. Anisotropic diffraction of wave 4 from this grating generates the extraordinarily (TE-) polarized wave 3, the phase-conjugate replica of wave 1. The grating recorded by the waves 3 and 4, $K = k_3 - k_4$, is identical to the initial grating. The grating period is determined by birefringence, $\Lambda = 2\pi/K = \lambda/(n_{\lambda^*} - n_{\epsilon^*})$, where $n_{\lambda^*}$ and $n_{\epsilon^*}$ are the effective refractive indices of the TM- and TE-polarized modes and $\lambda$ is the vacuum wavelength of light.

The electric field $E$ of each wave is separated into an amplitude $A$ and a normalized component of electric field (mode) $U$, such that $E(x,y) = A U(y)$. Here the longitudinal field component of the TM modes can be neglected. The coupling equations for two counterpropagating pairs of orthogonally

![Fig. 1. Interaction scheme of anisotropic four-wave mixing with $k_{1,2} = 2\pi n_{\lambda^*}/\lambda$ for the TE wave vector, $k_{2,4} = 2\pi n_{\epsilon^*}/\lambda$ for the TM wave vector, and $K = 2\pi/\Lambda$ for the grating vector.](image-url)
Fig. 2. Experimental arrangement for anisotropic four-wave mixing in a waveguide. M's, mirrors; BS's, beam splitters; PB, polarizing beam splitter; L's, microscope lenses; OF, optical fiber; S's, shutters; RS, rotary stage; PD's, photodetectors; W, waveguide.

Polarized waves can be written in the form:

\[ \frac{dA_1}{dx} = \sqrt{h_2/n_e} (\gamma A_2^2 A_1 - \gamma A_2 A_3 A_4) - \frac{\alpha_1}{2} A_1, \]

(2)

\[ \frac{dA_2}{dx} = \sqrt{h_2/n_e} (-\gamma A_1^2 A_2 + \gamma A_1 A_3 A_4) - \frac{\alpha_2}{2} A_2, \]

(3)

\[ \frac{dA_3}{dx} = \sqrt{h_2/n_e} (-\gamma A_4^2 A_3 + \gamma A_3 A_2 A_1) + \frac{\alpha_1}{2} A_3, \]

(4)

\[ \frac{dA_4}{dx} = \sqrt{h_2/n_e} (\gamma A_3^2 A_4 - \gamma A_3 A_1 A_2) - \frac{\alpha_2}{2} A_4, \]

(5)

with

\[ R(\gamma) = \frac{\omega \hbar \omega n_e^2}{2 \sigma} \eta r_{222} \beta_{222}, \]

(6)

\[ J(\gamma) = \frac{\omega \hbar \omega n_e^2}{2 \sigma} \eta r_{222} \beta_{222}, \]

(7)

\[ \eta = \frac{1}{\sqrt{h_1 h_2}} \int U_1^* U_2 U_2 dy. \]

(8)

Here \( h \) is the effective thickness of the TE and TM modes, according to the normalization \( \int U_1^* U_2 dy = h \delta_1 \). \( \sigma \) is the conductivity, \( \alpha \) is the absorption coefficient, and \( \omega \) and \( c \) are the frequency and the speed of light, respectively. We have assumed equal modes and absorption coefficients of the waves 1, 3 and 2, 4, respectively. The overlap of the interacting light fields with the space-charge field is expressed by the dimensionless factor \( \eta \). The spatial distribution of the space-charge field, \( U_{sc} \), follows from a differential equation that contains the modes \( U_{1,2,3,4} \).

The first part of the coupled equations (2)–(5) accounts for two-beam coupling of the beams 1, 2 and 3, 4. The parametric interaction of the beams 1 and 2 with the grating written by 3 and 4 (and vice versa) is described by the second part. In iron-doped LiNbO\(_3\) the amplification direction for two-beam coupling is from ordinarily to extraordinarily polarized waves, thus signal wave 1 and phase-conjugate wave 3 are amplified simultaneously owing to direct two-beam coupling and parametric mixing.

Waveguides are prepared by indiffusion of titanium (160-nm titanium film, annealed 56 h at 1000°C) into nominally pure \( \gamma \)-cut lithium niobate crystals. To enhance the photorefractive sensitivity, iron (80 nm) is indiffused additionally in the same manner as titanium. Owing to the large concentration of iron (approximately 0.7% at the surface) these waveguides exhibit high dark conductivity of the order of some \( 10^{-11} \text{ A V}^{-1} \text{ m}^{-1} \), considerably exceeding photoconductivity.

Figure 2 shows the experimental setup. We use light of the green line (514.5 nm) of an Ar\(^*\)-ion laser. The light is split into three different beams and coupled into polarization-conserving monomode fibers. Care has to be taken to obtain equal optical path lengths. Rutile prisms are used to couple radiation into and out of the waveguide. The input coupling efficiency is measured to be approximately 60%. We assume an output coupling efficiency of 100%, because the input losses that are due to the Gaussian beam do not occur. If the output coupling efficiency is smaller than 100%, the values determined for the reflectivity of the phase-conjugate signal are still larger than 65%. The beam size of the two pump beams and the signal beam are adjusted to be 0.4 mm in the waveguide, and the interaction length is 6.5 mm.

The time development of two- and four-wave interaction is shown in Fig. 3. During the time interval from \( t = 0 \) to \( t = 90 \) s, only the signal wave 1 and the pump wave 2 are switched on (two-beam coupling), whereas pump wave 4 is switched off. The signal wave is amplified up to a stationary value. At \( t = 90 \) s the second pump wave 4 is switched on, and additional amplification owing to parametric mixing occurs. The phase-conjugate (PC) wave now appears owing to diffraction of the second pump wave and is further amplified owing to the four-wave mixing.

![Fig. 3. Time development of two-wave (0 < t < 90 s) and four-wave (t > 90 s) interaction. Up to 90 s the signal wave is only amplified owing to two-beam coupling. At t = 90 s the second pump wave 4 is switched on, and the signal wave is amplified additionally owing to parametric mixing, thus a new stationary value is reached. When the pump wave 4 is switched on, the](image-url)
phase-conjugate wave appears owing to diffraction of the wave 4 from the grating written by the waves 1 and 2. The grating that is now written by the waves 3 and 4 leads to a further increase of the phase-conjugate wave power.

In Fig. 4 the dependence of signal and phase-conjugate wave power (TE\(_5\) modes) is shown as a function of the pump-wave power (TM\(_5\) modes). The input power of the signal beam is 26 µW, and the power of the two pump waves is equal. The symbols indicate the measured values, and the curves represent the corresponding solution of the coupled-wave equations (2)–(5). Here the values \(\alpha_{23} = 3.0 \times 10^{-13} \text{A/V}^2\) and \(\alpha_{32} = 4.9 \times 10^{-13} \text{A/V}^2\) obtained by measurements of two-beam coupling and diffraction efficiency are used. The outcoupled power of the signal beam is rather small because of the high absorption (\(\alpha = 350 \text{ m}^{-1}\)) in the waveguide. As can be seen, for low pump power the experimental values fit well to the theoretical dependence, whereas for higher pump power the theory predicts a stronger increase of both signal and phase-conjugate wave power. This can be explained by the following observation. At higher power levels, we find an increased scattering of the pump waves into the \(m\) lines of the excited TM modes. The beam spreading reduces the pump-wave intensity or the spatial overlap of the pump waves with the signal and phase-conjugate waves, respectively.

Because at the transition from the coupling prism to the waveguide and vice versa part of the phase information of the beams is lost, we did not succeed in testing the phase-conjugate wave by the use of a phase aberrator in the signal beam. Therefore we made the signal beam slightly convergent or divergent in the waveguide to test the phase conjugation. In the first case, we observe a divergent phase-conjugate beam, and in the second case, the phase-conjugate beam is convergent with a focus a few centimeters behind the outcoupling prism.

Owing to the high absorption in our waveguides, the output power of the phase-conjugate beam can be enlarged when the interaction length is adjusted to an optimum value. Calculations with the same input powers and waveguide properties as above, but a variable interaction length, show that the phase-conjugate reflectivity for an interaction length of 3.5 mm is twice that for 6.5 mm. In the experiment, it was not possible to reduce the interaction length because of the nonrectangular coupling prisms.

In conclusion, we have observed efficient anisotropic four-wave mixing in LiNbO\(_3\) waveguides with reflectivities of the phase-conjugate signal of as much as 65%. Comparison with the solution of the corresponding coupled-wave equations has shown that scattering in the waveguide limits the input pump power. Furthermore it should be possible to optimize the phase-conjugate reflectivity by reducing the interaction length in the waveguide.

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References